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PROPERTIES OF SELF SIMILAR SOLUTIONS OF REACTION-DIFFUSION SYSTEMS OF QUASILINEAR EQUATIONS

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ABSTRACT

In this paper, considered a parabolic system of two quasi linear reaction-diffusion equations with the source and absorption task and the properties of self-similar solutions of a system of quasi linear reaction-diffusion equations for the source and absorption task. Self-similar system of equations is constructed by the method of nonlinear splitting. Estimates of the solutions and the free boundary that arises in this case are found, which makes it possible to choose suitable initial approximations for each value of the numerical parameters.

***KEYWORDS**: Source and Absorption Equation, Parabolic System, Quasi Linear Equations, Self-Similar System of Equations & Reaction-Diffusion*

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Original Article

INTRODUCTION

Study of nonlinear mathematical models of various physical, biological, chemical and other phenomena and processes is one of the important directions of mathematical modeling. As examples, we note such physical theories as non-linear quantum mechanics, nonlinear electrodynamics and optics, non-linear theory of plasmas, nonlinear acoustics, nonlinear conduction, nonlinear diffusion, and other theories based on mathematical models which are nonlinear differential equations in partial derivatives. The study of the linear mathematical models of physical processes are easy to study, since the underlying linear differential equations developed general methods for their solution [1,2]. In applied tasks, the actual physical processes are nonlinear and for their adequate description, one should use nonlinear mathematical models.

Nonlinear models of mathematical physics, describing phenomena and processes in a wider range of physical parameter changes and have a better capacity of information about these phenomena and processes. Linear models are typically special cases of nonlinear models. They can give only an approximate picture of the phenomenon under study without identifying the observed effects. Studies show that the nonlinearities change not only the quantitative characteristics of the processes, but the qualitative picture of their behavior. Interestingly, from the point of view of applications to study the following classes of nonlinear differential equations in which the unknown function and the derivative of this function consists of exponential way. Then, with the comparison theorems of solutions of this class can be extended. These types of nonlinearities are often encountered in problems of the theory of filtration, diffusion, thermal conductivity, magnetohydrodynamics, biological populations, the oil industry, etc. [3,4,5,6].

These models more accurately describe the physics of the process and, therefore, their research shows that there are new effects related to the nonlinearity of the studied process. So was found the effects of the finite speed of propagation of perturbations [1], localization of solutions and different modes of processes. The first effect of finite speed of propagation of perturbations was obtained and applied to the problem of nonlinear thermal conductivity, in the work of Zeldovich and A. S. Kompaneetz[1], for the problem of nonlinear filtering in the work of G. I. Barenblatt[2,3], which are independently from each other, and got this effect. Further mathematical questions were studied in works of O. A. Oleinik, A. S. Kalashnikov [5,6,7], A. A. Samarskiy and his students [2,3-4], B. Knerr [3].

Statement of the Problem

In the field Q={(t,x): 0<t<∞, x∈R} considers a parabolic system of two quasilinear equations of source and absorption

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_1^{\sigma_1} \dfrac{\partial u_1}{\partial x} \right) + k_1(t) u_1 \cdot \left(1 - u_2^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_2^{\sigma_2} \dfrac{\partial u_2}{\partial x} \right) + k_2(t) u_2 \cdot \left(1 - u_1^{\beta_2}\right), \end{cases} \tag{1}$$

$$u_1\big|_{t=0} = u_{10}(x),\ u_2\big|_{t=0} = u_{20}(x), \tag{2}$$

Which describes the process of heat distribution in a nonlinear two-component environment, the diffusion coefficient of which is equal to $D_1 u_1^{\sigma_1}$ and $D_2 u_2^{\sigma_2}$, $\sigma_1, \sigma_2, \beta_1, \beta_2$ - positive real numbers, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - desired solution.

Estimates of solutions are found, which makes it possible to choose suitable initial approximations for each value of numerical parameters.

Construction of a Self-Similar System of Equations

Self-similar system of equations is constructed by the method of nonlinear splitting[4].

$$\begin{cases} \dfrac{d}{d\xi}\left(f_1^{\sigma_1} \dfrac{df_1}{d\xi}\right) + \dfrac{\xi}{2}\dfrac{df_1}{d\xi} + \mu_1\left(f_1 - f_1\ f_2^{\beta_1}\right) = 0, \\ \dfrac{d}{d\xi}\left(f_2^{\sigma_2} \dfrac{df_2}{d\xi}\right) + \dfrac{\xi}{2}\dfrac{df_2}{d\xi} + \mu_2\left(f_2 - y_2\ f_1^{\beta_2}\right) = 0, \end{cases}$$

where $\mu_i = \dfrac{1}{(1-\gamma_i \sigma_i)}$,

To construct the upper solution of the system (1) - (2), we introduce functions

$$\overline{f}_1(\xi) = A(a - \xi^2)_+^{n_1},\ \overline{f}_2(\xi) = B(a - \xi^2)_+^{n_2},$$

where $n_1 = -\dfrac{1}{\sigma_1}$, $n_2 = -\dfrac{1}{\sigma_2}$, $(b)_+ = \max(0, b)$,

$$u_1(t,x) \le u_{1+}(t,x) = e^{-\int_0^t k_1(\eta)d\eta}\,\overline{f}_1(\xi),$$

$$u_2(t,x) \le u_{2+}(t,x) = e^{-\int_0^t k_2(\eta)d\eta}\,\overline{f}_2(\xi),$$

$\overline{f}_1(\xi),\ \ \overline{f}_2(\xi)\ u\ \ \tau(t)$ -the functions defined above.

Cross-Diffusion System of Two Quasilinear Reaction-Diffusion Equations of the Source and Absorption

In the domain Q={(t,x): 0<t<∞, x∈R} considered a parabolic system of two quasilinear reaction-diffusion equations for the source and absorption

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 u_2^{\sigma_1}\dfrac{\partial u_2}{\partial x}\right) + k_1(t)u_1\left(1-u_2^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u_1^{\sigma_2}\dfrac{\partial u_1}{\partial x}\right) + k_2(t)u_2\left(1-u_1^{\beta_2}\right), \end{cases} \tag{3}$$

$u_1\big|_{t=0} = u_{10}(x)$, $u_2\big|_{t=0} = u_{20}(x)$,

which describes the process of process of absorption in a nonlinear two-component environment, diffusion coefficient whose are $D_1 u_2^{\sigma_1}$ and $D_2 u_1^{\sigma_2}$, $\sigma_1, \sigma_2, \beta_1, \beta_2$ - positive real numbers, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - sought solutions.

Let $\gamma_2\sigma_1 > 1$, $\gamma_1\sigma_2 = \gamma_2\sigma_1$, $\sigma_2(b_2+1) + \beta_2(b_1+1) = \sigma_1(b_1+1) + \beta_1(b_2+1)$, $c_i > 0$. In this case, assuming in (3)

$$w_i\,(\tau(t), x) = f_i(\xi), \quad \xi = |\,x\,| / \tau_1^{1/2},\ i = 1, 2,$$

and taking into account that the equation for $w_i(\tau, x)$ without lower terms always have a self-similar solution in the case $1-\gamma_{3-i}\sigma_i \ne 0,$ obtain the system

$$\begin{cases} \dfrac{d}{d\xi}\left(f_2^{\sigma_1}\dfrac{df_2}{d\xi}\right) + \dfrac{\xi}{2}\dfrac{df_2}{d\xi} + \mu_1\left(f_1 - f_1\ f_2^{\beta_1}\right) = 0, \\ \dfrac{d}{d\xi}\left(f_1^{\sigma_2}\dfrac{df_1}{d\xi}\right) + \dfrac{\xi}{2}\dfrac{df_1}{d\xi} + \mu_2\left(f_2 - f_2\ f_1^{\beta_2}\right) = 0, \end{cases} \tag{4}$$

where

$$\mu_i = \frac{1}{1-\gamma_{3-i}\sigma_i}.$$

To construct the upper solution of the system (4), we introduce functions

$$\overline{f}_1(\xi) = A(a - \xi^2)_+^{n_1}, \ \ \overline{f}_2(\xi) = B(a - \xi^2)_+^{n_2},$$

where

$$n_1 = \frac{(\sigma_1 + 2)}{1 - (\sigma_1 + 1)(\sigma_2 + 1)}, \ n_2 = \frac{(\sigma_2 + 2)}{1 - (\sigma_1 + 1)(\sigma_2 + 1)}, \ \ (b)_+ = \max(0, b),$$

$$\overline{f}_2^{\sigma_1} \frac{d\overline{f}_2}{d\xi} = -2B^{\sigma_1+1}\gamma_2 \xi \overline{f}_1 \in C(0,\infty),$$
$$\overline{f}_1^{\sigma_2} \frac{d\overline{f}_1}{d\xi} = -2A^{\sigma_2+1}\gamma_1 \xi \overline{f}_2 \in C(0,\infty)$$

and

$$\begin{cases} \dfrac{d}{d\xi}\left(\overline{f}_2^{\sigma_1} \dfrac{d\overline{f}_2}{d\xi} \right) = -2\gamma_2 B^{\sigma_1+1} \left(\overline{f}_1 + \xi \dfrac{d\overline{f}_1}{d\xi} \right), \\ \dfrac{d}{d\xi}\left(\overline{f}_1^{\sigma_2} \dfrac{d\overline{f}_1}{d\xi} \right) = -2\gamma_1 A^{\sigma_2+1} \left(\overline{f}_2 + \xi \dfrac{d\overline{f}_2}{d\xi} \right). \end{cases}$$

A and B are chosen from the system of nonlinear algebraic equations

$$\gamma_2 B^{\sigma_1+1} = 1/2,$$
$$\gamma_1 A^{\sigma_2+1} = 1/2,$$

and

$$\beta_1 = 1/n_2, \ \ \beta_2 = 1/n_1.$$

Then in the domain Q according to the principle of comparing solutions we have

$$u_1(t,x) \le u_{1+}(t,x) = e^{-\int_0^t k_1(\eta)d\eta} \overline{f}_1(\xi),$$

$$u_2(t,x) \le u_{2+}(t,x) = e^{-\int_0^t k_2(\eta)d\eta} \overline{f}_2(\xi),$$

where $\overline{f}_1(\xi), \ \overline{f}_2(\xi) \ and \ \ \tau(t)$ -previously defined function.

Slow diffusion.Case $n_1 > 0, n_2 > 0, \ n > 0$ **(slow diffusion).** Using the method of nonlinear splitting for the solution of the equation (4) obtained the following functions

$$\overline{\theta}_1(\xi) = (a - \xi^2)_+^{n_1}, \ \overline{\theta}_2(\xi) = (a - \xi^2)_+^{n_2},$$

where

$$n_1 = \frac{(\sigma_1+2)}{n},\ n_2 = \frac{(\sigma_2+2)}{n},\ n = 1-(\sigma_1+1)(\sigma_2+1),$$

$a>0, (y)_+ = \max(y,\ 0)$, $\xi < a$. It is known that for the global existence of a solution of the task (4) of the functions $f_i(\xi)$ must satisfy the following inequality:

$$\begin{cases} \dfrac{d}{d\xi}(f_2^{\sigma_1}\dfrac{df_2}{d\xi}) + \dfrac{\xi}{2}\dfrac{df_2}{d\xi} + \mu_1(f_1 - f_1\ f_2^{\beta_1}) \le 0, \\ \dfrac{d}{d\xi}(f_1^{\sigma_2}\dfrac{df_1}{d\xi}) + \dfrac{\xi}{2}\dfrac{df_1}{d\xi} + \mu_2(f_2 - f_2\ f_1^{\beta_2}) \le 0, \end{cases}$$

and

$$\beta_1 = 1/n_2,\ \beta_2 = 1/n_1.$$

It is shown that the functions $\bar{\theta}_1(\xi), \bar{\theta}_2(\xi)$ will be asymptotic finite solutions.

Теорема 1. Finite solution of the problem (4) at $\xi \to a_-$ has asymptotics $f_i(\xi) \sim \bar{\theta}_i(\xi)$, $i = 1,2$.

Fast diffusion.Case $n_1 > 0, n_2 > 0,\ n < 0$ **(fast diffusion).** For (4) there is

$$\chi_1(\xi) = (a+\xi^2)^{\ n_1},\ \chi_2(\xi) = (a+\xi^2)^{\ n_2},$$

where

$$n_1 = \frac{-(\sigma_1+2)}{n},\ n_2 = \frac{-(\sigma_2+2)}{n},\ n = (\sigma_1+1)(\sigma_2+1)-1,$$

$a>0$.

Theorem 2. At $\xi \to +\infty$ the vanishing at infinity solution of the problem (4) has asymptotic $f_i(\xi) \sim \chi_i(\xi)$.

And also in the domain Q={(t,x): 0<t<∞, x∈R} considered parabolic system of two quasilinear equations of the source and absorption task

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{11}u_1^m + a_{12}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{11}u_1^m + b_{12}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_1(t)u_1(1-u_2^{\beta_1}) \\ \dfrac{\partial u_2}{\partial t} = \dfrac{\partial}{\partial x}\left[(a_{21}u_1^m + a_{22}u_2^m)\dfrac{\partial u_1}{\partial x} + (b_{21}u_1^m + b_{22}u_2^m)\dfrac{\partial u_2}{\partial x}\right] + k_2(t)u_2(1-u_1^{\beta_2}) \end{cases} \quad (5)$$

a_{ij}, b_{ij} - positive real numbers, $\beta_1, \beta_2 \ge 0$, $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ - desired solution. At $a_{ij} \ne 0$, $b_{ij} = 0$ or $a_{ij} = 0$, $b_{ij} \ne 0$ mathematical model (5) is a system of the type reaction-diffusion with diffusion

coefficients $a_{ij}u_i^m \ge 0,\ b_{ij}u_i^m \ge 0$. In the case where at least one of the coefficients $a_{ij} \ne 0$ and $b_{ij} \ne 0$ (the sign can be arbitrary), system (5) is cross-diffusion (mutual-diffusion for i,j=1,2).

Qualitative properties of the considered problem are studied by constructing self-similar system of equations for (4).

Then, the obtained self-similar

system $$\begin{cases} \dfrac{d}{d\xi}\left[(a_{11}f_1^m + a_{12}f_2^m)\dfrac{df_1}{d\xi} + (b_{11}f_1^m + b_{12}f_2^m)\dfrac{df_2}{d\xi}\right] + \dfrac{\xi}{2}\dfrac{df_1}{d\xi} + \psi_1 f_1(1 - f_2^{\beta_1}) = 0\ , \\ \dfrac{d}{d\xi}\left[(a_{21}f_1^m + a_{22}f_2^m)\dfrac{df_1}{d\xi} + (b_{21}f_1^m + b_{22}f_2^m)\dfrac{df_2}{d\xi}\right] + \dfrac{\xi}{2}\dfrac{df_2}{d\xi} + \psi_2 f_2(1 - f_1^{\beta_2}) = 0, \end{cases}$$

(6)

and $\tau_1 = \tau_1(t)$ is chosen as

$$\tau_1(\tau) = \begin{cases} \dfrac{(\mathrm{T}+\tau)^{-\gamma_1 m+1}}{-\gamma_1 m+1}, & if \quad -\gamma_1 m + 1 \ne 0, \\ \ln(T+\tau), & if \quad -\gamma_1 m + 1 = 0, \\ (T+\tau), & if \quad m = 0, \end{cases}$$

if $\gamma_2 m_1 = \gamma_1 m_2$.

System (6) has an approximate solution of the form

$$\overline{f}_1 = A(a - b\xi^2)_+^{\eta_1},\quad \overline{f}_2 = B(a - b\xi^2)_+^{\eta_2} \quad (y)_+ = \max(0, y)\ .$$

Case I

$$\eta_1 = \eta_2,\ \eta_1 = \frac{1}{m},$$

and the coefficients A and B are determined from the solution of the systems of nonlinear algebraic equations

$$\begin{cases} \left[a_{11}A^m + a_{12}B^m\right]\cdot \eta_1 A + \left[b_{11}A^m + b_{12}B^m\right]\cdot \eta_2 B \cdot (-2b) = A, \\ \left[a_{21}A^m + a_{22}B^m\right]\cdot \eta_1 A + \left[b_{21}A^m + b_{22}B^m\right]\cdot \eta_2 B \cdot (-2b) = B. \end{cases}$$

Case II

$$\begin{aligned} &a_{11} = 0;\ a_{12} \ne 0;\ b_{11} = 0;\ b_{12} = 0; \\ &a_{21} = 0;\ a_{22} = 0;\ b_{21} \ne 0;\ b_{22} = 0; \\ &\eta_2 = \frac{1}{m}, \quad \eta_1 = \frac{1}{m}. \end{aligned}$$

And the coefficients A and B are determined from the solution of the system of nonlinear algebraic equations

$$a_{12}B^m \cdot \eta_1(-2b) = 1,$$

$$b_{21}A^m \cdot \eta_2(-2b) = 1.$$

Case III

$$a_{11} = 0;\ a_{12} = 0;\ b_{11} = 0;\ b_{12} \neq 0;$$
$$a_{21} \neq 0;\ a_{22} = 0;\ b_{21} = 0;\ b_{22} = 0;$$

$$\eta_1 = \frac{m+2}{(\mathrm{m+1})^2 - 1}, \qquad \eta_2 = \frac{m+2}{(\mathrm{m+1})^2 - 1}.$$

And the coefficients A and B are determined from the solution of the system of nonlinear algebraic equations

$$\begin{cases} b_{12}B^{m+1} \cdot \eta_2(-2b) = A, \\ a_{21}A^{m+1} \cdot \eta_1(-2b) = B. \end{cases}$$

Types of Self-Similar Regimes with Peaking

There can be three types of self-similar modes with peaking: NS, S, and LS. At $0 < \beta_i < m_i,\ i = 1,2$ HS mode is realized. Studies have shown that a self-similar problem in this case has a unique eigenfunction that decreases monotonically on a segment with a maximum at the center of symmetry. A self-similar solution is a wave whose amplitude and front increase in the blast mode.

At $\beta_i = m_i,\ i = 1,2$ takes place S−mode. Self-similar solution is a non-stationary dissipative localized structure. Inside the area of localization, the number of individuals increases in regime with aggravation, and beyond remains equal to zero.

Self-similar solution in LS−mode, is a non-stationary dissipative structure; all points of which move to the center of symmetry, the solution at $T = -\tau$ turns into infinity in a single point – the center of symmetry. Self-similar solutions can exist when $\beta_i > m_i,\ i = 1,2.$

The following are the results of numerical experiments for different values of the parameters.

To determine the number of eigenfunctions (EF) self-similar problem in LS − mode, and studies of the nature of their dependence on the parameter was conducted bifurcation analysis of the solutions. The first ten eigenfunctions continued on the parameter β_i for a fixed parameter m_i ($m_i = 2$).

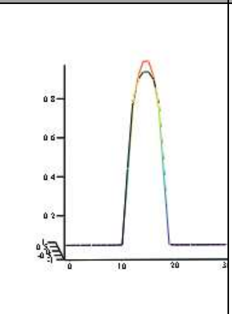
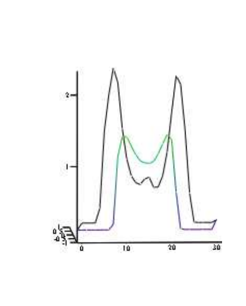
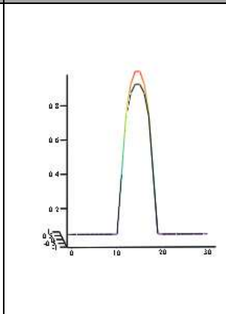
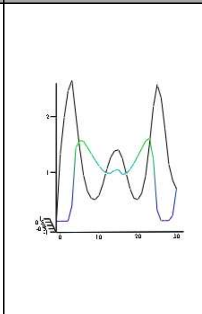
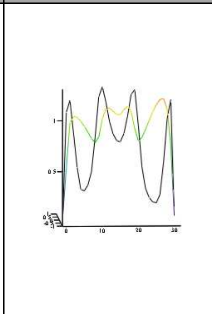
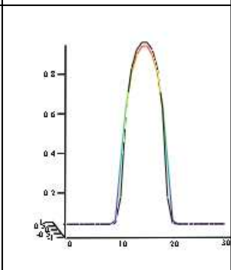
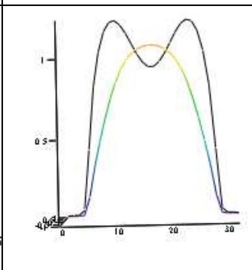
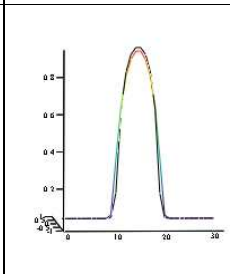
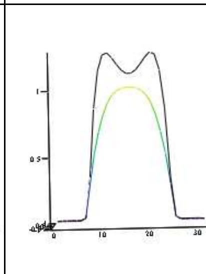
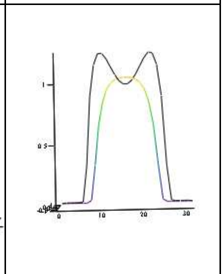
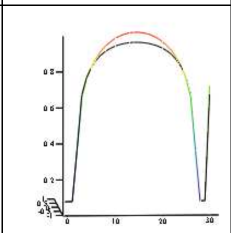
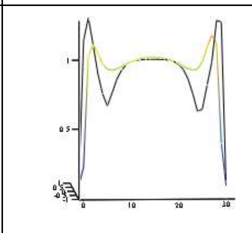
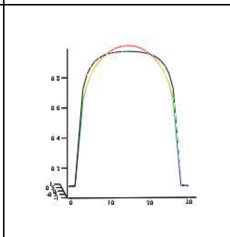
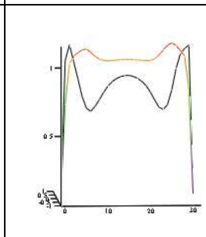
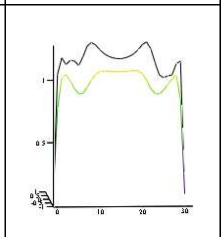
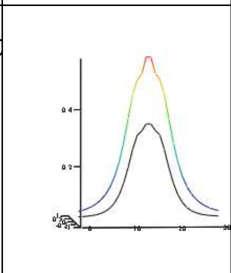
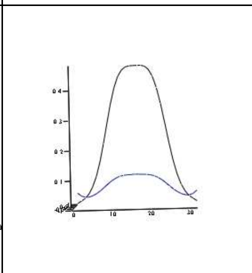
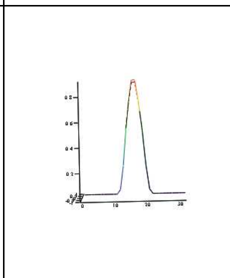
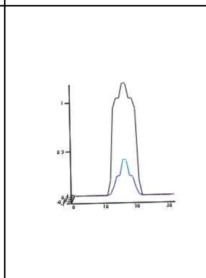
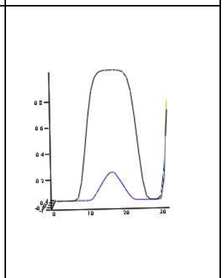
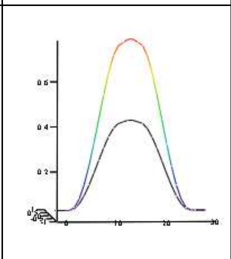
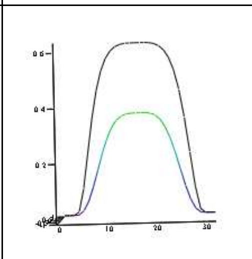
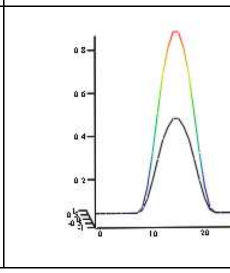
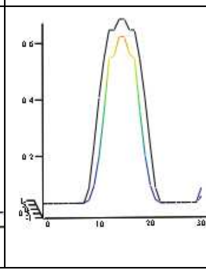
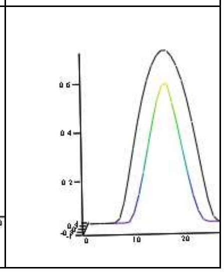
Parameter Values	$t=1$ $a_{11}=1; a_{12}=1; b_{11}=1; b_{12}=1$ $a_{21}=1; a_{22}=1; b_{21}=1; b_{22}=1$	$t=40$ $a_{11}=1; a_{12}=1; b_{11}=1; b_{12}=1$ $a_{21}=1; a_{22}=1; b_{21}=1; b_{22}=1$	$t=1$ $a_{11}=0; a_{12}=1; b_{11}=0; b_{12}=0$ $a_{21}=0; a_{22}=0; b_{21}=1; b_{22}=0$	$t=40$ $a_{11}=0; a_{12}=1; b_{11}=0; b_{12}=0$ $a_{21}=0; a_{22}=0; b_{21}=1; b_{22}=0$	$t=40$ $a_{11}=0; a_{12}=0; b_{11}=0; b_{12}=1$ $a_{21}=1; a_{22}=0; b_{21}=0; b_{22}=0$
LS-regime $m_1=2$, $m_2=2$ $\beta_1=3,\ \ k_1=2$ $\beta_2=3,\ \ k_2=9$ $eps=10^{-3}$ $m_i<\beta_i<\hat{\beta}_{ij}$ $\hat{\beta}_{ij}=m_i+m_i/(j-1)$, $j=3$					
S-regime $m_1=2$, $m_2=2$ $\beta_1=2,\ \ k_1=2$ $\beta_2=2,\ \ k_2=9$ $eps=10^{-3}$ $\beta_i=m_i$					
S-regime $m_1=4$, $m_2=4$ $\beta_1=4,\ \ k_1=2$ $\beta_2=4,\ \ k_2=9$ $eps=10^{-3}$ $m_i=\beta_i$					
HS-regime $m_1=0.7$, $m_2=0.7$ $\beta_1=0.5,\ \ k_1=2$ $\beta_2=0.5,\ \ k_2=3$ $eps=10^{-3}$ $\beta_i<m_i$					
HS-regime $m_1=1.4$, $m_2=1.4$ $\beta_1=0.5,\ \ k_1=2$ $\beta_2=0.7,\ \ k_2=5$ $eps=10^{-3}$ $\beta_i<m_i$					

Figure 1: Results of Computational Experiment

CONCLUSIONS

The analysis showed that the eigenfunction with the number $j=2,3,4,\ldots$ exists in the interval $m_i<\beta_i<\hat{\beta}_{ij}$, where

$$\hat{\beta}_{ij} = m_i + m_i /(j-1)\,,\; i = 1,2\,,\;\; j = 2,3,4,... \qquad (7)$$

Values $\beta_i = m_i$ и $\beta_i = \hat{\beta}_{ij}$ are the bifurcation points in which the EF ceases to exist. The first EF exists for any value $\beta_i > m_i$, $i = 1,2$. It follows from (7) that when $\beta_i > \hat{\beta}_{i2} = 2m_i$, $i = 1,2$ self-similar task in LS mode can have only one eigen function.

The greater the number of SF, the narrower the interval of the parameter β_i in which it exists.

The number of eigenfunctions N that has a self-similar problem for the given β_i and m_i is determined by the formula, where $a_i = \beta_i /(\beta_i - m_i)$, $i = 1,2$:

$$N_i = \begin{cases} [a_i], if \;\; a \; is \; not \; an \text{ integer}, \\ a_i - 1, \; if \;\; a \; is \text{ integer}. \end{cases}$$

The sequence of bifurcation points that define the right boundary of the domain of existence of eigenfunctions is an infinite sequence that converges to a point 1 + = ∞ σβ, which is the common left boundary of the intervals of existence of all eigenfunctions in the LS-mode.

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